

**THE CONDITIONAL RELATION BETWEEN HIGHER-ORDER
SYSTEMATIC
CO-MOMENTS AND RETURNS IN INDIAN EQUITY MARKET**

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Abstract

Two of the important conditions for the traditional CAPM to hold is that the expected market risk premium must be positive and that the security returns must follow the normal distribution. However, the validity of the traditional CAPM model is tested on realized returns rather than on expected return and the realized returns may be positive or negative. Further, many times the security returns do not follow normal distribution. The current study aims at developing the model which incorporates higher moments (co-skewness and co-kurtosis) and also incorporates both rising and declining market in the same model. The model is tested for the Indian equity market. The results of the study describing the conditional relationship between co-moments and return show that only beta and co-skewness are priced in the Indian market and not the co-kurtosis. The results further indicate the asymmetric relationship between betas and return in up and down markets and symmetric relationship between co-skewness and return in up and down markets.

Keywords: Capital Asset Pricing Model; Conditional Relationship; Higher-Order Moments; Emerging Market; Indian Stock Market.

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1. Introduction

The traditional Capital Asset Pricing Model (CAPM) developed by Sharpe (1964), Lintner (1965) and Mossin (1966) assumes that security returns are normally distributed and the expected market risk premium is always positive. The traditional CAPM model assumes that only the first two moments (mean and variance) of security returns explain the expected returns variation. In the traditional CAPM, investors assume the quadratic utility function of their wealth. However, if security returns do not follow normal distribution and/or utility function of investors is non-quadratic, then the assumption that mean and variance of returns are the only determinants of investors' choice cannot be justified. Rubinstein (1973) argued that in case of non-quadratic utility function of investors and non-normal security returns distribution, the quantification of risk requires higher moments like skewness and kurtosis in addition to variance. Kraus and Litzenberger (1976) extended the traditional CAPM model by incorporating co-skewness in the model. According to Arrow (1965), to incorporate the first three moments of security returns in the selection of risky portfolios by an investor, the utility function of the investor should include the following three properties: (i) the marginal utility of wealth is positive; (ii) marginal utility of wealth is declining (risk aversion); and (iii) absolute risk-aversion is non-increasing function of wealth. Skewness of security returns measures the degree of asymmetry of security returns distribution around the mean security return. If the skewness of security return is positive (negative) it means that the distribution of security returns has a long tail towards the right (left) or positive (negative) side of the distribution of security returns. Investors prefer positive skewness of security returns to negative skewness of security returns. If the skewness of returns is negative, the investor will expect higher returns to compensate for the additional risk involved in the negative skewness of security returns. On the other hand, if the skewness of returns is positive, the investor will expect the lower returns for the same variance as he or she may be willing to forego some returns for the reduction in risk because of the presence of positive skewness in security returns. When the skewness of individual or portfolio security returns are jointly analyzed with the skewness of market returns, it is called as systematic skewness or co-skewness.

The empirical findings from studies such as Dittmar (2002) further extended the CAPM model by incorporating co-kurtosis in the model. In addition to the three properties (positive marginal utility, risk aversion and non-increasing absolute risk aversion) mentioned above, he also

included decreasing absolute risk prudence in the utility function of the investor. Kurtosis of security returns measures the shape of the probability distribution of security returns. If the security returns follow normal distribution, the kurtosis of security returns is three. The kurtosis is generally defined in terms of excess kurtosis (that is, in excess of kurtosis of normal distribution or three). If the excess kurtosis of security returns is positive, it means that the tail of the distribution of security returns has a heavier tail and has a higher peak than the normal distribution. On the other hand, if the excess kurtosis of security returns is negative, it means that tail of security return distribution is lighter and flatter than the normal distribution. To be more precise, kurtosis of security returns measures the probability of extreme security returns. When the kurtosis of individual or portfolio security returns are jointly analyzed with the kurtosis of market returns, it is called as systematic kurtosis or co-kurtosis.

Many theoretical and/or empirical researches have analyzed the impact of co-skewness and/or co-kurtosis on the stock return [Arditti (1967); Barone-Adesi (1986); Chunachinda, et al. (1997); Ingersoll (1975); Jurczenko and Maiilet (2001, 2006); Rubinstein (1973); Sears and Wei (1985); Tan (1991); Chiao, et al. (2003); Kraus and Litzenberger (1976); Harvey and Siddiqui (2000); Dittmar (2002); Friend and Wasterfield (1980); Peiro (1999); Doan, Lin and Zurbruezz (2010); Fang and Lai (1997); Lim (1989); Smith (2006); Christie and Chaudry (2001); Conrad, et al. (2008); Doan, et al. (2014); Fletcher and Kihanda(2005); Galagedera, et al. (2004); Hung, et al. (2003); Teplova and Shutova (2011)].

In this paper, co-skewness and co-kurtosis are incorporated in the traditional CAPM model of Sharpe (1964), Lintner (1965) and Mossin (1966). The empirical literature shows that testing on four moment CAPM model has been done more recently (since the late 1990s). Dittmar (2002), Fang and Lai (1997), Conrad, et al. (2008) and Doan, et al. (2010) have examined the impact of co-skewness and co-kurtosis on stock returns for the US market. Fletcher and Kihanda (2005) and Hung, et al. (2003) have examined the importance of co-skewness and co-kurtosis in explaining the variation in the stock returns for the UK stock market. Galagedera, et al. and Doan, et al. (2010) examine the importance of higher moments (skewness and kurtosis) for the Australian stock market. Chiao, et al. (2003) tested the four-moment CAPM model for Taiwan stock market. Teplova and Shutova (2011) and Silva (2005) analyze the impact of co-skewness

and co-kurtosis on stock returns for the Russian stock market and Brazilian stock market respectively.

We believe that researchers should critically examine the assumptions and limitation of the traditional CAPM model before empirically testing the validity of the model. The traditional CAPM model assumes that the security returns follow normal distribution and the expected market risk premium is positive. However, the empirical validity of the traditional CAPM is tested on the realized returns (rather than on expected returns) and on the assumption that security returns follow normal distribution. Many times the realized market risk premiums are not positive and/or security returns do not follow normal distribution. Since equity is a long-term source of investment, in the long run we may assume that expected market risk premium is positive. However, in the short run we may not always expect that market risk premium is positive. Thus, we believe that the traditional CAPM describes the long-run relationship between market risk and expected return where the expected market risk premium is always positive. Here, we suggest a model which describes both short-run and long-run relationship between beta and expected return; co-skewness and expected return; and co-kurtosis and expected return. That is, the model describes the direction and intensity of impact of beta, co-skewness and co-kurtosis on security return when the expectation about the market is positive (expected market risk premium is positive), and describes what kind of relationship between beta and return; co-skewness and return; and co-kurtosis and return is expected when the expectation about the market is negative (expected market risk premium is negative). This is what Pettengill, Sundaram and Mathur (1995) call conditional relationship between market risk and return. According to Pettengill, Sundaram and Mathur (1995), if realized return are taken as a proxy for expected return, there is a (an) direct (inverse) relationship between beta and return if the realized market risk premium is positive (negative). This approach of investigating the relationship between market risk (co-variance risk, co-skewness risk and co-kurtosis risk) and return further helps us to assess whether relationship between market risk and return is symmetric during negative and positive markets.

The main objective of this paper is to assess the conditional relationship between beta and return; conditional relationship between co-skewness and return; and conditional relationship between co-kurtosis and return during up and down markets separately in the Indian stock market. The up

market has been defined as realized market return being more than the risk free rate (that is, market risk premium is positive) and down market has been defined as realized market return being less than the risk-free rate of return (that is, market risk premium is negative). This paper further investigates whether the relationships between beta and return; co-skewness and return; and co-kurtosis and return are symmetric during up and down markets. The testing of symmetry of beta during up and down market is relevant and may make additional contribution in the existing literature of asset pricing as the traditional CAPM assumes that beta is symmetric during up and down market.

There are few studies which have investigated the conditional relationship between co-moments and return [Friend and Wasterfield (1980); Chiao, et al. (2003); Galagedera, et al. (2004); Tang and Shum (2003); and Teplova and Shutova (2011)].

As per CAPM, ex-ante, the market returns cannot be less than risk free rate. However, in actual market, the ex-post realized market return may be less than the risk free rate which may result in non-existence of relationship between risk and return as predicted by the traditional CAPM. The presence of many negative market excess return periods implies that earlier studies were prejudiced against searching a systematic relationship while analyzing for an unconditional relationship between co-moments and realized returns. Further, Zhang and Wihlborg (2010) have emphasized that in emerging markets, where periods with negative realized market excess returns are expected to be observed repeatedly, distinction between up and down markets is essential for the study of relationship between co-moments and returns.

In the light of the fact that negative realized excess market returns are observed frequently in the emerging markets like India, the aim of the current study is to analyze the relationship between co-moments and returns during the up and the down markets separately in the Indian Equity Market. Unlike previous studies in the context of Indian Equity Market not supporting the applicability of traditional CAPM, this study, to the best of our knowledge, is the first work to account for the conditional relationship between higher co-moments and return in the Indian Equity market. Most of the asset pricing models have been empirically tested in the context of developed economies. In the context of emerging economies, this topic has not been researched

intensively. Since the contextual framework of emerging economies may be significantly different from the developed economies, the results of this study may further contribute to the existing literature of asset pricing models.

The rest of the paper is organized as follows. Section 2 explains the empirical model. Section 3 deals with methodology and data base of the study. Section 4 analyzes the empirical results respectively while Section 5 gives concluding remarks.

2. Empirical Model

The current study uses the pooled regression model to assess the relationship between co-moments (co-variance, co-skewness and co-kurtosis) and returns for both up and down market and thus it does not require to use Fama and Macbeth (1973) approach. This study empirically tests both unconditional and conditional relationship between co-moments and return for the Indian market.

To test the unconditional relationship between co-moments and return the following empirical regression model has been used.

$$r_{pt} - r_{ft} = a_0 + a_1\beta_p + a_2\gamma_p + a_3\delta_p + u'_{pt}, p = 1, 2, \dots, n; t = T_1, T_2, \dots, T_T; \text{ and } T_1 > T \quad (1)$$

Where,

$$\beta_p = \frac{\sum_{t=1}^T (r_{it} - \bar{r}_i)(r_{mt} - \bar{r}_m)}{\sum_{t=1}^T (r_{mt} - \bar{r}_m)^2} \quad (2)$$

$$\gamma_p = \frac{\sum_{t=1}^T (r_{it} - \bar{r}_i)(r_{mt} - \bar{r}_m)^2}{\sum_{t=1}^T (r_{mt} - \bar{r}_m)^3} \quad (3)$$

$$\delta_p = \frac{\sum_{t=1}^T (r_{it} - \bar{r}_i)(r_{mt} - \bar{r}_m)^3}{\sum_{t=1}^T (r_{mt} - \bar{r}_m)^4} \quad (4)$$

Beta (β_p), co-skewness (γ_p), and co-kurtosis (δ_p) of the portfolio are estimated from expressions (2), (3) and (4) respectively. The regression model (1) which is the pooled regression model tests the relationship between co-moments and return using the beta, co-skewness and co-kurtosis estimated (2), (3) and (4) respectively. The betas, co-skewness and co-kurtosis are estimated for

each security and then assigned to portfolios for the first time period (five- year period), and for the next time period (next five- year period), the relationship between co-moments and return is assessed using the co-moments estimated from the first time period. There is no overlapping between the two time periods. The first period is called co-moments estimation period and the second period is called testing period.

In the model (1), we expect the positive sign of a_1 , negative sign of a_2 , and positive sign of a_3 . If beta, co-skewness and co-kurtosis are considered as a measurement of market risk, there must be a situation in which a portfolio which has a higher beta, lower co-skewness, and higher co-kurtosis portfolios must earn the rate of return lower than a portfolio which has a lower beta, higher co-skewness and lower co-kurtosis otherwise no investor will invest in low beta, high co-skewness and low kurtosis portfolios. The relationship between co-moments and return depends upon the relationship between realized rate of market return and risk-free rate of return. If the realized rate of market return is more than the risk-free rate of return, there is a direct relationship between beta and return (that is, high beta portfolios will earn the rate of return higher than the low beta portfolios), inverse relationship between co-skewness and return and direct relationship between co-kurtosis and return. However, if the realized rate of market return is less than the risk-free, there is an inverse relationship between beta and return (that is, high beta portfolios will earn the rate of return lower than the low beta portfolios), direct relationship between co-skewness and return and inverse relationship between co-kurtosis and return. Thus, the traditional higher moments CAPM model needs to be modified which incorporates the conditions of both up market and down market in the same model. This is what we call the conditional relationship between co-moments and return. To estimate the conditional relationship between co-moments and return, co-moments estimation method as specified in equation (2), (3) and (4) remains the same.

To estimate the conditional relationship between co-moments and return, the following testing model has been used.

$$r_{pt} - r_{ft} = a_0 + a_1\beta_p + a_2D\beta_p + a_3\gamma_p + a_4D\gamma_p + a_5\delta_p + a_6D\delta_p + u'_{pt}, p = 1, 2, \dots, n; t = T_1, T_2, \dots, T_T; \text{ and } T_1 > T \quad (5)$$

Where:

$D = 1$, if the realized market return is less than the risk-free rate; and

$D = 0$, if the realized market return is more than the risk-free rate.

As mentioned above, there is a (an) direct (inverse) relationship between beta and return during up (down) market; an (a) inverse (direct) relationship between co-skewness and return during up (down) market; and a (an) direct (inverse) relationship between co-kurtosis and return during up (down) market. Thus, we expect positive sign of estimated coefficient of a_1 , negative sign of estimate coefficient of $a_1 + a_2$, negative sign of estimated coefficient of a_3 , positive sign coefficient of $a_3 + a_4$, positive sign of estimated coefficient of a_5 , and negative sign of estimated coefficient of $a_5 + a_6$. That is, we expect negative sign of estimated coefficient of a_2 , positive sign of estimated coefficient of a_4 , and negative sign of estimated coefficient of a_6 . We further expect that the absolute values of coefficients of a_2 , a_4 and a_6 are greater than the absolute value of estimated coefficient of a_1 , a_3 and a_5 respectively.

The current study also tests for the symmetric relationship between beta and return; co-skewness and return; and co-kurtosis and return during up and down markets. This test will help us in assessing whether the impact of beta, co-skewness and co-kurtosis on realized return during up market is more or less than the impact of beta, co-skewness and co-kurtosis on realized return during down market.

To test for symmetry of beta and finding out in which kind of market the impact of beta on return is more, the following hypothesis has been formulated.

Null Hypothesis $H_0: 2a_1 + a_2 = 0$, against

Alternative hypothesis $H_a: 2a_1 + a_2 \neq 0$

If the null hypothesis accepted, it means that there is symmetric relationship between beta and return during up and down market. However, if the null hypothesis is rejected in favour of an alternative hypothesis, it means the relationship between beta and return is not symmetric. The t-

statistic has been used to test for the symmetric relationship between beta and return. If value of t-statistic comes out to be positive (negative) and significant, it means that the impact of beta on returns is higher during up (down) market than during down (up) market.

To test for symmetry of co-skewness and finding out in which kind of market the impact of beta on return is more, the following hypothesis has been formulated.

Null Hypothesis $H_0: 2a_3 + a_4 = 0$, against

Alternative hypothesis $H_a: 2a_3 + a_4 \neq 0$

If the null hypothesis accepted, it means that there is symmetric relationship between co-skewness and return during up and down market. However, if the null hypothesis is rejected in favor of an alternative hypothesis, it means the relationship between co-skewness and return is not symmetric. If value of t-statistic comes out to be negative (positive) and significant, it means that the impact of co-skewness on returns is higher during up (down) market than during down (up) market.

To test for symmetry of co-kurtosis and finding out in which kind of market the impact of co-kurtosis on return is more, the following hypothesis has been formulated.

Null Hypothesis $H_0: 2a_5 + a_6 = 0$, against

Alternative hypothesis $H_a: 2a_5 + a_6 \neq 0$

If the null hypothesis accepted, it means that there is symmetric relationship between co-kurtosis and return during up and down market. However, if the null hypothesis is rejected in favour of an alternative hypothesis, it means the relationship between co-kurtosis and return is not symmetric. If value of t-statistic comes out to be positive (negative) and significant, it means that the impact of co-kurtosis on returns is higher during up (down) market than during down (up) market.

The above empirical models have been tested for the Indian stock market. The data base, methodology and analysis of empirical results of the current study are described in the following sections.

3.Data and Methodology

The study presented herein is representative from the period April, 2000 to March, 2015. The sample consists of the stocks in S&P BSE 500 Index. Out of the total population of 500 companies in the S&P BSE 500 Index, 270 company's monthly stock prices data were available for the entire sample period i.e. April, 2000 to March, 2015. In addition, there were 80 more companies whose monthly stock price data were available from April, 2005 to March, 2015. As a result, the research comprises of 270 companies for the period from April, 2000 to March, 2005 and 350 companies from April, 2005 to March, 2015. Out of the major stock exchanges in India i.e. Bombay Stock Exchange (BSE) and National Stock Exchange (NSE), the former is the oldest and has larger number of companies listed. To add to this, most of the sampled companies are also listed on NSE. Additionally, it may be noted that there is negligible price difference of the securities in these two exchanges as trading in these two exchanges is done through electronic mode.

The Stock price returns are calculated using the formula:

$$r_{it} = \ln\left(\frac{P_{it}}{P_{i,t-1}}\right)$$

Where,

r_{it} = Return on stock i .

P_{it} = Price per share of stock i at the end of the month t .

$P_{i, t-1}$ = Price per share of stock i at the end of the month $t-1$.

The S&P BSE 500 index, a value weighted index, has been taken as proxy for market portfolio. It covers all major industries of the Indian Economy. It represents nearly 93% of the total market capitalization of total number of stocks listed in Bombay Stock Exchange (BSE).

The market returns are calculated as:

$$r_{mt} = \ln\left(\frac{P_{mt}}{P_{m,t-1}}\right)$$

Where,

r_{mt} = Monthly return on the market portfolio

P_{mt} = Value of the S&P BSE 500 Index at the end of the month t .

$P_{m,t-1}$ = Value of the S&P BSE 500 Index at the end of the month $t-1$.

The above data of all the sample stocks and index was obtained from Prowess, online database maintained by Centre for Monitoring of Indian Economy (CMIE). The 91-days treasury bill rates (which has been taken as a proxy for the risk free rate) has been taken from the official website of Reserve Bank of India (RBI). Since in the RBI database the Treasury Bill Rates are quoted on annual basis, these rates are converted into monthly equivalents as per the following formula

$$r_{ft} = \sqrt[12]{1 + TBR} - 1$$

Where,

r_{ft} = Monthly rate of return on the risk-free asset.

TBR = Annual rate of return on 91-day Treasury Bills

The period from April 2000 to March 2015 reveals that 91- days T-Bill Rate exceeds the market return in 77 out of 180 total observations(42.77%).The presence of a large number of negative market excess return periods may result in non-existence of relationship between risk and return as predicted by the traditional CAPM.Thus the objective of the paper is to test for the conditional relationship between the co-moments and realized returns.

The research has been carried out using the following steps:

- For the market index (S&P BSE 500) and each of the stocks, monthly returns through natural logarithm of price relatives were calculated. Further, the excess stock returns and excess market returns were calculated.

- This was followed by estimating beta, co-skewness and co-kurtosis for each of the stocks on the basis of equation (2), (3) and (4) mentioned above respectively. Beta, co-skewness and co-kurtosis have been estimated on the period of 5 years' data and then tested on next five year data. First of all, the beta, co-skewness and co-kurtosis have been estimated from April 2000 to March 2005 and then tested for the time period from April 2005 to March 2010. Similarly, betas, co-skewness and co-kurtosis are estimated from April 2005 to March 2010 and tested for the time period from April 2010 to March 2015 respectively.
- The testing of systematic, conditional relationship between co-moments and realized returns was carried through pooled regression analysis on the portfolios formed. The portfolios were formed based on beta, co-skewness and co-kurtosis. For beta sorted portfolios, the stocks were arranged in descending order of beta and grouped into 30 portfolios. (10 High beta portfolios, 10 Medium Beta Portfolios, and 10 low beta portfolios) Each portfolio is constructed such that portfolio 1 contains stocks representing highest beta values and the last portfolio representing lowest beta values. Similarly, the stocks were arranged separately in descending order of co-skewness and co-kurtosis and the similar approach was followed for the construction of co-skewness and co-kurtosis sorted portfolios. Thus, in total 90 (30 each for beta sorted portfolios, co-skewness sorted portfolios and co-kurtosis portfolios) portfolios were constructed for the time period from April 2000 to March 2005 and for the time period from April 2005 to March 2010. This was done to achieve diversification and thus reduce any error that might occur due to the presence of unsystematic risk as done in Amanullah and Kamaiah (1998).

4.Data Analysis & Findings

The descriptive statistics about BSE-500 index (a broad stock market index of the Indian stock market) is given in Table 1. The mean, standard deviation, skewness and excess kurtosis of returns have been computed for each of the individual securities and BSE-500 index. In addition to these statistics, the Jarque-Bera test has been conducted to test the normality of returns of each of the individual securities.

The skewness and excess kurtosis have been computed as follows:

$$Skewness = \frac{1}{n} \sum_{t=1}^n \left[\frac{r_{it} - \bar{r}}{\sigma_i} \right]^3$$

$$\text{Excess Kurtosis} = \frac{1}{n} \sum_{t=1}^n \left[\frac{r_{it} - \bar{r}}{\sigma_i} \right]^4$$

Where:

r_{it} : is the return of the security/index during the time period t.

σ_i : is the standard deviation of the rate of return of the security/index.

n: is the number of observations.

\bar{r} : is the mean return of the security/index

The excess kurtosis here means the kurtosis of the rate of returns of the portfolio over the above the return of the normal distribution (if return of the portfolio follows the normal distribution, the kurtosis of the rate of returns of the portfolio is three). The Jarque-Bera (JB) test uses the following statistic to test the normality.

$$JB = n \left[\frac{S^2}{6} + \frac{(K - 3)^2}{24} \right]$$

Where n is the sample size, S is the skewness coefficient of the return of the portfolio and K is the kurtosis coefficient of the rate of return of the portfolio. If the returns of the portfolio follow the normal distribution, S is equal to 0 and K is equal to 3. Thus, the Jarque-Bera test of the normality of the returns of the portfolio is similar to the test of the joint hypothesis that skewness coefficient and kurtosis coefficients of the returns of the portfolio are zero and three respectively. Under this hypothesis, we expect the value of JB statistic to be zero. For large sample, the JB statistic follows the χ^2 (chi-square) distribution with two degrees of freedom. If the calculated probability value of the JB statistics is adequately small (that is, the value of the JB statistic is adequately high), the hypothesis that the returns of the portfolio are normally distributed can be rejected. On the other hand if the calculated probability value of the JB statistics is adequately high (that is, the value of the JB statistic is adequately low), the hypothesis that the returns of the portfolio are normally distributed is not rejected.

Table 1: Descriptive Statistics of BSE-500

	Mean	Median	Skew	Kurtosis	Count	JB Test
BSE-500	0.0042	0.0113	-0.7123	2.3280	180	55.8663

The results of the Table 1 show, that returns of BSE-500 index (a broad index of the Indian equity market) are generally asymmetric and leptokurtic. The mean coefficient of skewness for the S&P BSE 500 index has come out to be -0.712. The average excess kurtosis for the S&P BSE 500 index is 2.32. The Jarque-Bera test of normality for the BSE-500 index shows that the returns of BSE-500 index exhibit significant non-normality at 1% level.

The regression models specified in the data and methodology section has been estimated to assess the unconditional and conditional relationship between co-moments and realized returns. The results describing the unconditional (conditional) relationship between co-moments and returns are shown in Table 2-Table 5 (Table 6-Table 9). The pooled regression equations describing the relationship between co-moments and returns have been estimated for the portfolios (the construction of which was described in the previous section). First, the impact of beta and co-skewness on realized returns has been studied by estimating the regression model with beta and co-skewness as explanatory variables and realized returns as dependent variable. Thereafter, co-kurtosis was also included as explanatory variable in addition to beta and co-skewness in the regression model to assess the impact of beta, co-skewness and co-kurtosis on realized returns. This was done to investigate whether both co-skewness and co-kurtosis are priced in the Indian stock market or only one of them or none of them. Before testing the conditional approach, we have also tested unconditional higher moment CAPM to assess whether the unconditional higher moment CAPM holds in the Indian Equity Market.

Table 2:

Beta, Skewness and Kurtosis estimated from April 2000 to March 2005. Pooled Regression Analysis on Portfolio data for the period April 2005 to March 2010. (Unconditional Three Moment Model)

Portfolios	a_0 (Intercept)	a_1 (Beta)	a_2 (Skewness)	\bar{R}^2
Beta Sorted Portfolio	0.0095 (0.9228)	0.0042 (0.1676)	-0.0036 (-0.1883)	-0.0011
Skewness Sorted Portfolio	-0.0008 (-0.0446)	0.0207 (0.6397)	-0.0099 (-0.6084)	-0.0009
Kurtosis Sorted Portfolio	0.0080 (0.5827)	0.0062 (0.2154)	-0.0041 (-0.2209)	-0.0011
Combined Total	0.0077 (1.0690)	0.0065 (0.4513)	-0.0041 (-0.4487)	-0.0003

Note Figures in () indicate the value of t-statistics

Table 3:

Beta, Skewness and Kurtosis estimated from April 2005 to March 2010. Pooled Regression Analysis on Portfolio data for the period April 2010 to March 2015. (Unconditional Three Moment Model)

Portfolios	a_0 (Intercept)	a_1 (Beta)	a_2 (Skewness)	\bar{R}^2
Beta Sorted Portfolio	0.0210 (4.1288)*	-0.0055 (-0.4841)	-0.0087 (-0.7828)	0.0043
Skewness Sorted Portfolio	0.0196 (1.5162)	-0.0149 (-0.7219)	0.0023 (0.2342)	0.0003
Kurtosis Sorted Portfolio	0.0209 (3.8586)*	-0.0190 (-1.4965)	0.0052 (0.4674)	0.0035
Combined Total	0.0201 (5.8986)*	-0.0143 (-2.2899)**	0.0012 (0.2562)	0.0033

Note Figures in () indicate the value of t-statistics

*Significant at 1% level

**Significant at 5% level

Table 4:

Beta, Skewness and Kurtosis estimated from April 2000 to March 2005. Pooled Regression Analysis on Portfolio data for the period April 2005 to March 2010.(Unconditional Four Moment Model)

Portfolios	a_0 (Intercept)	a_1 (Beta)	a_2 (Skewness)	a_3 (Kurtosis)	\bar{R}^2
Beta Sorted Portfolio	0.0108 (0.9274)	0.0151 (0.2895)	0.0043 (0.1136)	-0.0200 (-0.2386)	-0.0016
Skewness Sorted Portfolio	-0.0007 (-0.0375)	0.0232 (0.4106)	-0.0087 (-0.3222)	-0.0039 (-0.0550)	-0.0014
Kurtosis Sorted Portfolio	0.0080 (0.5800)	0.0030 (0.0557)	-0.0062 (-0.1814)	0.0054 (0.0735)	-0.0016
Combined Total	0.0079 (1.0738)	0.0094 (0.3264)	-0.0024 (-0.1458)	-0.0047 (-0.1176)	-0.0005

Note Figures in () indicate the value of t-statistics

Table 5

Beta, Skewness and Kurtosis estimated from April 2005 to March 2010. Pooled Regression Analysis on Portfolio data for the period April 2010 to March 2015. (Unconditional Four Moment Model)

Portfolios	a_0 (Intercept)	a_1 (Beta)	a_2 (Skewness)	a_3 (Kurtosis)	\bar{R}^2
Beta Sorted Portfolio	0.0205 (3.2827)*	-0.0006 (-0.0165)	-0.0082 (-0.6864)	-0.0048 (-0.1381)	0.0038

Skewness Sorted Portfolio	0.0246 (1.5602)	-0.0431 (-0.7840)	-0.0002 (-0.0179)	0.0255 (0.5533)	-0.0000
Kurtosis Sorted Portfolio	0.0154 (1.3623)	0.0067 (0.1399)	0.0092 (0.6947)	-0.0240 (-0.5548)	0.0031
Combined Total	0.0198 (4.0758)*	-0.0126 (-0.5452)	0.0014 (0.2590)	-0.0017 (-0.0779)	0.0031

Note Figures in () indicate the value of t-statistics

*Significant at 1% level

The results shown in Tables 2 and 3 describe the unconditional relationship between returns and beta; and co-skewness and return for the portfolios constructed on the basis of sorting of beta, co-skewness and co-kurtosis. Table 2 (Table 3) shows the results pertaining to co-moments (beta and co-skewness) estimated for the portfolios from April, 2000 to March, 2005 (April, 2005 to March, 2010) and tested for the unconditional relationship of these co-moments (betas and co-skewness) with returns of the portfolios pertaining to time period from April, 2005 to March, 2010 (April, 2010 to March, 2015). The coefficients of beta and co-skewness generally have not come out to be significant in Table 2 and Table 3. This contradicts the traditional theory of three-moments CAPM that there is a positive long term relationship between beta and expected returns and an inverse relationship between co-skewness and expected returns. Thus the results describing the unconditional relationship show that the unconditional three-moments CAPM does not hold for both the time periods in Indian Equity Market.

The results shown in Tables 4 and 5 describe the unconditional relationship between returns and beta; co-skewness and return; and co-kurtosis and returns for the portfolios constructed on the basis of sorting of beta, co-skewness and co-kurtosis. Table 4 (Table 5) shows the results pertaining to beta, co-skewness and co-kurtosis estimated for the portfolios from April, 2000 to March, 2005 (April, 2005 to March, 2010) and tested for the unconditional relationship of these betas, co-skewness and co-kurtosis with returns of the portfolios pertaining to time period from April, 2005 to March, 2010 (April, 2010 to March, 2015). The coefficients of beta, co-skewness and co-kurtosis have not come out to be significant in Table 4 and Table 5. This contradicts the

traditional theory of four moments CAPM that there is a positive long term relationship between beta and expected returns, an inverse relationship between co-skewness and expected returns, and direct relationship between co-kurtosis and returns. Thus the results describing the unconditional relationship show that the unconditional four-moments CAPM does not hold for both the time periods in Indian Equity Market.

The main reason for the insignificant unconditional relationship between co-moments and realized returns may be that significant relationship between co-moments and returns holds only if the excess market returns during the period of the study of the relationship is significantly positive. However, during the time period covered in this study, the excess market return is not positively significant.

Table 6:

Beta, Skewness and Kurtosis estimated from April 2000 to March 2005. Pooled Regression Analysis on Portfolio data for the period April 2005 to March 2010. (Conditional Three Moment Model)

Portfolios	a_0 (Intercept)	a_1 (Beta)	a_2 ($D*\beta_p$)	a_3 (Skewness)	a_4 ($D*\gamma_p$)	\overline{R}^2
Beta Sorted Portfolio	0.0095 (1.2934)	0.1286 (6.7401)*	-0.3395 (-17.3978)*	-0.0742 (-4.7548)*	0.1927 (9.1122)*	0.4905
Skewness Sorted Portfolio	-0.0008 (-0.0628)	0.1220 (5.2195)*	-0.2764 (-22.6289)*	-0.0518 (-4.1756)*	0.1143 (9.3676)*	0.4953
Kurtosis Sorted Portfolio	0.0080 (0.8205)	0.1196 (5.5937)*	-0.3091 (-19.6982)*	-0.0608 (-4.1787)*	0.1546 (9.4219)*	0.4950
Combined Total	0.0077 (1.5015)	0.1148 (10.7252)*	-0.2954 (-34.6220)*	-0.0551 (-7.6428)*	0.1393 (15.6783)*	0.4930

Note Figures in () indicate the value of t-statistics

*Significant at 1% level

Table 7:

Beta, Skewness and Kurtosis estimated from April 2005 to March 2010. Pooled Regression Analysis on Portfolio data for the period April 2010 to March 2015. (Conditional Three Moment Model)

Portfolios	a_0 (Intercept)	a_1 (Beta)	a_2 ($D*\beta_p$)	a_3 (Skewness)	a_4 ($D*\gamma_p$)	\overline{R}^2
BetaSorted Portfolio	0.0210 (5.4193)*	0.0351 (2.9924)*	-0.0839 (-5.0732)*	-0.0107 (-0.9039)	0.0040 (0.2355)	0.4221
Skewness Sorted Portfolio	0.0196 (1.9908)**	0.0379 (2.3271)**	-0.1093 (-12.5055)*	-0.0114 (-1.3384)	0.0283 (3.4196)*	0.4202
Kurtosis Sorted Portfolio	0.0209 (5.0753)*	0.0328 (2.6240)*	-0.1071 (-6.5128)*	-0.0078 (-0.6738)	0.0269 (1.6255)	0.4240
Combined Total	0.0201 (7.7495)*	0.0359 (6.1903)*	-0.1038 (-15.1989)*	-0.0102 (-2.1166)**	0.0237 (3.4940)*	0.4226

Note Figures in () indicate the value of t-statistics

*Significant at 1% level

**Significant at 5% level

Table 8:

Beta, Skewness and Kurtosis estimated from April 2000 to March 2005. Pooled Regression Analysis on Portfolio data for the period April 2005 to March 2010. (Conditional Four Moment Model)

Portfolios	a_0 (Intercept)	a_1 (Beta)	a_2 ($D*\beta_p$)	a_3 (Skewness)	a_4 ($D*\gamma_p$)	a_5 (Kurtosis)	a_6 ($D*\delta_p$)	\bar{R}^2
Beta Sorted Portfolio	0.0108 (1.3015)	0.0766 (1.6382)	-0.1677 (-2.1732)* *	-0.0957 (-3.0780)*	0.2728 (6.6986) *	0.0727 (1.0070)	-0.2528 (-2.3007)* *	0.4915
Skewness Sorted Portfolio	-0.0007 (-0.0529)	0.1256 (2.5857) *	-0.2793 (-3.7476)*	-0.0502 (-2.2359)**	0.1131 (3.5827) *	-0.0054 (-0.0856)	0.0041 (0.0394)	0.4947
Kurtosis Sorted Portfolio	0.0080 (0.8167)	0.0861 (1.8960)	-0.2268 (-3.2980)*	-0.0818 (-2.7737)*	0.2062 (4.5783) *	0.0539 (0.8268)	-0.1325 (-1.2297)	0.4949
Combined Total	0.0079 (1.5084)	0.0988 (3.8900) *	-0.2438 (-6.0020)*	-0.0624 (-4.4432)*	0.1636 (7.9058) *	0.0231 (0.6472)	-0.0758 (-1.2978)	0.4930

Note Figures in () indicate the value of t-statistics

*Significant at 1% level

**Significant at 5% level

Table 9:

Beta, Skewness and Kurtosis estimated from April 2005 to March 2010. Pooled Regression Analysis on Portfolio data for the period April 2010 to March 2015. (Conditional Four Moment Model)

Portfolios	a_0 (Intercept)	a_1 (Beta)	a_2 ($D*\beta_p$)	a_3 (Skewness)	a_4 ($D*\gamma_p$)	a_5 (Kurtosis)	a_6 ($D*\delta_p$)	\bar{R}^2
Beta Sorted Portfolio	0.0205 (4.3096)*	0.0662 (1.8763)	-0.1382 (-3.1313)*	-0.0075 (-0.6082)	-0.0013 (-0.0775)	-0.0327 (-0.9629)	0.0577 (1.3257)	0.4220
Skewness Sorted Portfolio	0.0246 (2.0482)**	0.0164 (0.3431)	-0.1231 (-2.6032)*	-0.0120 (-1.0879)	0.0245 (1.6068)	0.0172 (0.3845)	0.0171 (0.2968)	0.4197
Kurtosis Sorted Portfolio	0.0154 (1.7921)	0.0726 (1.8496)	-0.1363 (-4.5662)*	0.0008 (0.0569)	0.0175 (0.9475)	-0.0419 (-1.1554)	0.0370 (1.1729)	0.4240
Combined Total	0.0198 (5.3561)*	0.0551 (2.7450)*	-0.1401 (-6.9510)*	-0.0057 (-0.9895)	0.0147 (1.7951)	-0.0228 (-1.1590)	0.0436 (1.9153)	0.4227

Note Figures in () indicate the value of t-statistics

*Significant at 1% level

**Significant at 5% level

The pooled regression equation(5) describing the conditional relationships between beta and returns, co-skewness and returns and co-kurtosis and returns have been estimated for the constructed portfolios (the construction of which was described in the previous section). The results shown in Tables 6 and 7 describe the conditional relationship between returns and beta, and co-skewness and returns. Table 6 (Table 7) shows the results pertaining to beta and co-skewness estimated from April, 2000 to March, 2005 (April, 2005 to March, 2010) and tested for the conditional relationship of these betas and co-skewness with returns pertaining to time period from April, 2005 to March, 2010 (April, 2010 to March, 2015). The results describing the conditional relationship for various beta, skewness and kurtosis sorted portfolios show that there exists a significant and direct relationship between beta and realized returns of portfolios during the up market whereas significant and inverse relationship between beta and return during the down markets, for both the time periods. Further an inverse relationship between co-skewness and returns during the up market and a direct relationship between co-skewness and returns during the down market, for both the time periods. Overall, the coefficients of beta and co-skewness have come to be significant in all the time periods both for up markets and down markets. All the coefficients also had the predicted signs.

The results shown in Tables 8 and 9 describe the conditional relationship between returns and beta, co-skewness and returns and co-kurtosis and returns. Table 8 (Table 9) shows the results pertaining to beta, co-skewness and co-kurtosis estimated from April, 2000 to March, 2005 (April, 2005 to March, 2010) and tested for the conditional relationship of these betas, co-skewness and co-kurtosis with returns pertaining to time period from April, 2005 to March, 2010 (April, 2010 to March, 2015). The results describing the conditional relationship between co-moments and realized returns show that after including the co-kurtosis (in addition to beta and co-skewness) in the model, the explanatory power of the regression model describing the conditional relationship between co-moments and realized returns has not increased in both the time periods. The coefficients of co-kurtosis have generally not come out to be significant both during the up market and down market and also for both the periods. Even the significance level of some of the coefficients of beta and co-skewness has been disturbed after including the co-kurtosis.

Thus, the overall results show that when both up and down market are incorporated separately in the model, only beta and co-skewness are priced in the Indian stock market and not the co-kurtosis. Thus, we can argue that the conditional three-moments CAPM model holds in the Indian market and not the conditional four-moments CAPM model. Further, the current study also tests for symmetry of beta and co-skewness during up and down markets. The t-test has been used to assess the symmetry of beta and co-skewness during up and down market. The results show that the absolute value of the coefficient of beta during the down market is significantly higher than the absolute value of the coefficient of beta during the up market whereas the co-skewness is found to be symmetric during the up and down market. (Table 10).

Table 10

Test for symmetry between co-moments and returns in up and down markets

Time Period	Regression Period April 2005 to March 2010		Regression Period April 2010 to March 2015	
	T-Statistics $2a_1 + a_2$	T-Statistics $2a_3 + a_4$	T-Statistics $2a_1 + a_2$	T-Statistics $2a_3 + a_4$
All Portfolios (Combined)	-2.8556*	1.7140	-2.3867**	0.2698

*Significant at 1% level

**Significant at 5% level

5-Conclusion:

The capital asset pricing model (CAPM) developed by Sharper (1964), Lintner (1965) and Mossin (1966) is one of the most important contributions in the field of financial economics. The traditional CAPM explains that the systematic risk of a security is only the relevant factor in explaining the expected returns of the security. However, except a few earlier studies (Fama and Macbeth, 1973), most of the studies have empirically rejected the validity of the traditional CAPM. Two of the most important requirements of the traditional CAPM to hold is that (i) excess market returns during the period of the study of this relationship must be positively significant; and (ii) security returns should follow the normal distribution. CAPM is considered as the one of the most important models by financial analysts despite of its many limitations. The empirical validity of the model is examined on the realized returns (rather than on the expected returns) and many times the realized market returns are negative. Further, in reality, security returns do not follow normal distribution. Thus, it requires some modifications in the traditional CAPM to assess the relationship between market risk and returns which incorporates higher moments (co-skewness and co-kurtosis) and also both rising market (realized return is more than the risk-free rate) and declining market (realized rate of return is less than the risk-free rate) in the same model. This is what is called conditional relationship between co-moments and return. The main objective of this paper is to assess the relationship between co-moments and return during up and down markets separately in the emerging market like India. The current study uses the approach adopted by Pettengill, Sundaram and Mathur (1995) with certain modifications.

The conditional relationship between co-moments and return has been studied for the portfolios constructed on the basis of sorting of individual securities according to beta, co-skewness and co-kurtosis. To empirically assess the conditional relationship between co-moments and realized return, the current study uses the data of the Indian stock market covering the time period from April, 2000 to March, 2015.

The empirical results of the current study indicate that only beta and co-skewness are priced in the Indian market. The results show that co-kurtosis is not priced in the Indian market. The results show there exists a significant and direct relationship between beta and realized returns of individual securities during the up market and significant, an inverse relationship between beta and return during the down markets, an inverse relationship between co-skewness and returns during the up market and a direct relationship between co-skewness and returns during the down market, for both the time periods. The coefficients of co-kurtosis have come out to be insignificant in both up and down markets. Thus, overall results show that only beta and co-skewness are priced in the Indian market and not the co-kurtosis.

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